Distributed Cochannel Interference Control in Cellular Radio Systems

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Abstract— **Optimum transmitter power control schemes to minimize the outage probability due to cochannel interference have been investigated in [5], [6]. The main drawback of the previously proposed algorithms is that they would require reliable measurements of the gains in all radio paths in the system. In this paper, distributed power control algorithms that use only the signal-to-interference (C/I) ratios in those links actually in use, are investigated. An algorithm, that successfully approximates the behavior of the best known algorithms is proposed. The algorithm involves a novel distributed C/I-balancing scheme. Numerical results show that capacity gains in the order of 3–4 times can be reached also with these distributed schemes. Further, the effects of imperfect C/I estimates due to noise, vehicle mobility, and fast multipath fading are considered. Results show that the balancing procedure is very robust to measurement noise, in particular if C/I requirements are low or moderate. However, for required high C/I levels or for a rapidly changing path loss matrix, convergence may be too slow to achieve substantial capacity improvements.**

I. Introduction

In the design of large high-capacity cellular radio systems, cochannel interference caused by frequency reuse is the single most limiting factor on the system capacity. Transmitter power control schemes have been proposed to control this interference for a given channel allocation. The main idea is to adjust the transmitter power in each base-mobile link such that the interference in other receiver locations is minimized. Maintaining sufficient transmission quality in the actual link is an obvious constraint.

Most of the early work in power control schemes has been focused on algorithms that aim at keeping the received power of the desired signal at some constant level. This has the favorable effect that the requirements on the receiver dynamic range are smaller, which results in better adjacent channel protection. Results from previous simulation studies generally indicate an increase of capacity by roughly a factor of two compared to systems with fixed transmitter power. Analytical investigations, however, show that constant-received power control has only limited ability to reduce *cochannel* interference. In [1], Aein provides an analytical approach to the problem. In his work concerning frequency reuse interference in satellite systems, he introduces the concept of *C/I balancing*. This scheme is devised to achieve the same C/I in all communication links. The balancing concept is successfully used by Nettleton and Alavi [2] in the context of cellular radio in general and spread-spectrum systems in particular. In [5], [6] the balancing concept of Aein, Nettleton, and Alavi is further refined and used to construct power control procedures that are optimal in the sense that they minimize the *interference probability* (or outage probability). The latter quantity is the probability of having a too low carrier-to-interference (C/I) ratio on a given link. The results, however, depend on full knowledge of the gain (or attenuation) in all propagation paths, both intended base-mobile paths and unwanted, interference paths. Therefore, the optimum algorithm described in [5], [6] serves mainly as a tool to derive upper bounds on the performance of interference control schemes, rather than being suited for actual implementation.

For practical implementation, we would have to rely on power control schemes that would require far less measurements and allow distributed operation. The implications of such distributed power control schemes have been investigated in a preliminary simulation study by Axén [3], [4]. He found that a simple proportional control algorithm, which increases the transmitter power in a link if the C/I level is too low and decreases the power when the C/I is more than adequate, will work well in most cases. However, some cases with instability were observed when the required "target" C/I was set too high. In these cases most of the transmitter power levels were "locked" at the highest possible output power. In the present paper, we will see that a novel distributed C/I-balancing scheme based on the results in [1], [2], [5], [6] may be used to modify the simple proportional control scheme to result in a stable, distributed algorithm. In the sequel we will also study the effects of different practical limitations such as limited knowledge of path-gains and path-gain estimation errors.

II. System Model and Previous Results

The model assumptions used throughout the paper are identical to the assumptions in [6]. We will therefore limit the presentation to a review of the essential details. Throughout the paper we will study a large, but finite, cellular radio system. To each active mobile-base pair we have allocated a pair of independent(orthogonal) channels (time slots, codes) for the up- (mobile-to-base) and down- (base-to-mobile) links. In the following we will focus on the cochannel interference, and assume that all adjacent channel interference and other noise sources can be neglected.

Now, let us focus on the set of those cells in which a particular channel pair m is used at some particular instant of time. The number of cells in this set, the *cochannel set* of

Manuscript received October 3, 1991; revised November 20, 1991 and February 6, 1992.

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IEEE Log Number 9201034.

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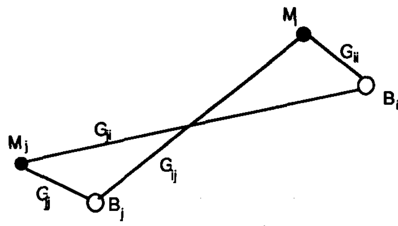

Fig. 1. System geometry and link gains.

m, is denoted by $Q = Q(m)$. Q will depend on the particular channel allocation scheme and the instantaneous traffic load. In each cell in this set only one transmitter–receiver pair is active on our particular channel pair. Since up- and down-link channels are assumed not to interfere with each other, all relevant propagation effects are modeled by the link gains in Fig. 1, where G_{ij} denotes the (power) gain from the base in cell j to the mobile using this channel in cell i. Note that the gains G_{ii} correspond to the desired communication links, whereas the G_{ij}, $i \neq j$, correspond to unwanted interference links. Further, we may note that in general $G_{ij} \neq G_{ji}$. For the time being we will assume that the instantaneous path gains are random variables. We will assume the transmission quality to be dependent only on the carrier-to-interference ratio (C/I), Γ, experienced by the receiver. The total interference power is modeled as the sum of the powers of all active interferers [8], [9]. In the following we will consider the interference situation in the *down-link* (base-to-mobile) path. The *C/I at mobile i* can now be expressed as

$$\Gamma_i = \frac{G_{ii}P_i}{\sum_{j\neq i} G_{ij}P_j} = \frac{P_i}{\sum_{j\neq i} P_j \frac{G_{ij}}{G_{ii}}} = \frac{P_i}{\sum_{j=1}^{Q} P_j Z_{ij} - P_i} \tag{1}$$

where we have introduced the (base station) *power vector* $\boldsymbol{P} = \{P_i\}$. The normalized *down-link gain matrix* $Z = \{Z_{ij}\}$ is defined as

$$Z_{ij} = \frac{G_{ij}}{G_{ii}}. \tag{2}$$

Since the path gains G_{ij} are random variables, the matrix Z will have random entries. In the sequel, we will be concerned with the (nonpathologic) case where Z and all (square) submatrices of Z are irreducible with probability one. This holds for all reasonable random propagation models and in particular for the "standard" model used in the numerical results section. Further, we will use the vector notation $\boldsymbol{\Gamma} = \{\Gamma_i\}$ to denote the C/I's in the down-links defined in (1). It is obvious that the corresponding up-link results can be derived by simply reversing the indices and letting $\boldsymbol{P}$ denote the powers of the mobile transmitters.

Due to the random nature of G and Z, the component Γ_i will be a random variable. Now, let Γ denote the C/I at some randomly chosen mobile. As our performance measure we will throughout the paper use the *interference (or outage) probability*, defined as

$$\begin{aligned} F(\gamma_0) &= \Pr\{\Gamma \leq \gamma_0\} \\ &= \sum_{j=1}^{Q} \Pr\{\Gamma \leq \gamma_0 | \text{mobile } j\} \Pr\{\text{mobile } j\} \\ &= \frac{1}{Q}\sum_{j=1}^{Q} \Pr\{\Gamma_j \leq \gamma_0\} \end{aligned} \tag{3}$$

where γ_0 is the minimum required (threshold) carrier-to-interference ratio or the system *protection ratio.* A carrier-to-interference ratio γ is defined to be *achievable* in the cochannel set, if there exists a power vector $\boldsymbol{P}$ such that $\Gamma_i \geq \gamma$ in all cells of the set. An important result is that the largest achievable carrier-to-interference ratio γ^* is related to the spectral properties of the matrix Z as [6]

$$\gamma^* = \frac{1}{\lambda^* - 1} \tag{4}$$

where λ^* is the largest (dominant) real eigenvalue of the matrix Z. The power vector $\boldsymbol{P}^*$ achieving this maximum was found to be the eigenvector of Z corresponding to the eigenvalue λ^*. $\boldsymbol{P}^*$ actually achieves the same carrier-to-interference ratio γ^* in all mobiles. We call such a system *balanced* [1], [2]. In [6] it was further demonstrated that one, somewhat surprising way, to minimize the interference probability given in (3) was to construct smaller and smaller balanced systems by *removing* cells (i.e., by turning their transmitters off). This procedure is repeated until the maximum achievable C/I, γ^*, of the remaining system is larger than the required protection ratio γ_0. Mathematically stated, the problem is to find the largest submatrix of Z with an eigenvalue λ^* smaller than λ_0, i.e.

$$\lambda^* \leq \lambda_0 = \frac{1 + \gamma_0}{\gamma_0}. \tag{5}$$

Given full knowledge of the matrix elements, finding an optimum power vector is a straightforward but very tedious task. An optimum power vector $\boldsymbol{P}'$ derived by the algorithm sketched above, would consist of a subvector of nonzero transmitter powers, corresponding to a C/I-balanced system. The rest of the vector would be zeros, corresponding to "removed" cells. "Removing" a cell from the cochannel set, which may seem like a drastic measure, does not necessarily result in the loss of a call since usually an intracell handoff to some other channel assigned to the base station is possible.

The preceding model assumptions provide an instantaneous system description for some given instant of time. However, since most of the mobiles in a cellular radio system are in motion, we will have to assume that the matrix Z for our channel set, in fact, is a stochastic process. In addition, due to changing traffic conditions also the dimension of Z will change. For the time being, however, we will assume that Z is changing slowly compared to the dynamics of our algorithms. Thus for our purposes, we may consider Z to be constant. We will return to the dynamic properties of Z in Section V.

III. Distributed C/I Balancing

Measuring all path gains, in real time, in a large cellular system would be a formidable task. Instead, we would be interested in what may be achieved with less explicit knowledge about the gain matrix Z. Now let us for this purpose consider a *discrete time power control algorithm* (DPCA), Ψ, that uses only the C/I in those base-mobile links that are actually in use. We could write

$$P^{(\nu+1)} = \Psi\left(P^{(\nu)}, \Gamma^{(\nu)}\right) \tag{6}$$

where the superscript ν denotes time. In such an algorithm, base stations and mobiles measure the C/I and report these measurements to some central network controller. The controller would then distribute power control decision throughout the network. On the other hand, a *distributed* DPCA, Ψ_D, may be defined to operate on each component of P as

$$P_i^{(\nu+1)} = \Psi_D\left(P_i^{(\nu)}, \Gamma_i^{(\nu)}\right) \tag{7}$$

and may thus control the transmitter power of the mobiles and their corresponding base station without involving some central controller. Here, only C/I measurements in the cell itself would affect the transmitter power in the cell. Now, let a *positive vector* P be a vector with all positive components (denoted $P > 0$). Consider the following algorithm.

DB Algorithm (Distributed Balancing Algorithm)

$$P^{(0)} = P_0, \qquad P_0 > 0$$

$$P_i^{(\nu+1)} = \beta P_i^{(\nu)}\left(1 + \frac{1}{\Gamma_i^{(\nu)}}\right), \qquad \beta > 0.$$

$\Diamond$

The algorithm starts with an arbitrary positive vector P_0. The C/I level Γ_i is measured in each link. Since this measurement (in the down-link) has to be made at the mobile, the result has to be reported back to the base station. The base station transmitter power is then adjusted according to the expression above. For the DB algorithm we have the following result:

Proposition: Using the DB algorithm, the system will approach C/I balance with probability one, i.e.,

$$\lim_{\nu\to\infty} P^{(\nu)} = P^*$$

$$\lim_{\nu\to\infty} \Gamma_i^{(\nu)} = \gamma^*.$$

Proof: Using (1) to rewrite the iteration yields

$$\begin{aligned} P_i^{(\nu+1)} &= \beta P_i^{(\nu)}\left(1 + \frac{1}{\Gamma_i^{(\nu)}}\right) \\ &= \beta P_i^{(\nu)}\left(1 + \frac{\sum_{j=1}^{Q} Z_{ij}P_j^{(\nu)} - P_i^{(\nu)}}{P_i^{(\nu)}}\right) \\ &= \beta \sum_{j=1}^{Q} Z_{ij}P_j^{(\nu)}. \end{aligned}$$

In vector form this can be expressed as

$$P^{(\nu+1)} = \beta Z P^{(\nu)}. \tag{8}$$

Now let $\lambda_1, \lambda_2, \lambda_3, \cdots, \lambda_Q$ be the eigenvalues of Z ordered such that

$$|\lambda_1| \geq |\lambda_2| \geq |\lambda_3| \geq \cdots |\lambda_Q|$$

and let $e_1, e_2, e_3, \cdots, e_Q$ be the corresponding (nonzero) eigenvectors. Since matrix Z is assumed to have full rank, the eigenvectors are distinct and linearly independent. We may thus write the start vector P_0 as

$$P_0 = c_1e_1 + c_2e_2 + c_3e_3 + \cdots + c_Qe_Q. \tag{9}$$

Now, iterating (8) we get [7]

$$\begin{aligned} P^{\nu} &= \beta^{\nu} Z^{\nu} P_0 \\ &= \beta^{\nu}\left(c_1\lambda_1^{\nu}e_1 + c_2\lambda_2^{\nu}e_2 + c_3\lambda_3^{\nu}e_3 + \cdots + c_Q\lambda_Q^{\nu}e_Q\right). \end{aligned} \tag{10}$$

As long as $c_1 \neq 0$, the latter expression will approach

$$P^{(\nu)} \to \beta^{\nu}c_1\lambda_1^{\nu}e_1 = c_1(\beta\lambda^*)^{\nu}P^* \tag{11}$$

since $\lambda^*(=\lambda_1)$ is the largest eigenvalue and $e_1(=P^*)$ the corresponding eigenvector. To prove that $c_1 \neq 0$, finally, we consider the transpose of Z, denoted Z^T. This matrix is positive and has the same eigenvalues as Z. Consider now, the eigenvectors of Z^T, denoted $e_1', e_2', e_3', \cdots, e_Q'$, ordered in the manner as above ($e_i' \neq 0$). Now multiplying (9) from the left by $(e_1')^T$ we obtain

$$\begin{aligned} (e_1')^T P_0 &= c_1(e_1')^T e_1 + c_2(e_1')^T e_2 + \cdots + c_Q(e_1')^T e_Q \\ &= c_1(e_1')^T e_1' + 0 + \cdots + 0 \end{aligned}$$

since $(e_1')^T e_j = 0$ for all $i \neq j$ [7]. Because e_1' corresponds to the maximum eigenvalue of the positive matrix Z^T, e_1' is certainly positive. We get

$$c_1 = \frac{(e_1')^T P_0}{(e_1')^T e_1} > 0$$

since all vectors are positive. This ensures proper convergence in (10).

$\Diamond$

The convergence properties of the iteration (8) are, in fact, well known from numerical analysis. The iteration procedure is essentially the *power method* (!) for finding the dominant eigenvalue and its corresponding eigenvector [10] of a matrix. The speed of convergence of the algorithm depends on how much larger λ_1 is than λ_2. The remaining error is approximately proportional to the magnitude of the quotient

$$\left|\frac{\lambda_2}{\lambda_1}\right|^{\nu}.$$

Since, however, our matrix Z has diagonal elements equal to one we may write

$$\mathrm{Tr}\,[\mathbf{Z}] = \sum_{j=1}^{Q} \lambda_j = Q.$$

Now for a high achievable C/I, λ^* will be close to one. We have that

$$\sum_{j=2}^{Q} \lambda_j = Q - \lambda^* \approx Q - 1. \tag{12}$$

Since the magnitude of all eigenvalue has to be less (or equal) to $|\lambda^*|$, (12) suggests that $|\lambda_j| \approx 1$ for all j's. This, in turn, results in slow convergence.

On the other hand, the choice of initial vector $\boldsymbol{P}_0$ is not very critical. Since we may

show [7] that $\boldsymbol{P}^*$ is the *only* positive eigenvector of $\boldsymbol{Z}$, almost any positive start vector will be reasonably "close" to $\boldsymbol{P}^*$ (i.e., have a large coefficient c_1 in (10)).

A practical problem is that the transmitter powers in the DB algorithm are all increasing, unless we choose the parameter β in a proper way. Ideally, selecting

$$\beta = \beta(\nu) = \frac{1}{\left[\boldsymbol{P}^{(\nu)}\right]} \tag{13}$$

would ensure a "constant" average power level. However, calculating this quantity may not be possible in a completely distributed system since it would require knowledge about the power levels in all links. This could be a topic for further research.

IV. Stepwise Removal with Distributed Balancing

In [6] it was shown that C/I balancing in combination with "cell removals" could be used to minimize the outage probability (3). Further, a simple algorithm, the stepwise removal algorithm (SRA), was introduced. This is a procedure of (roughly) linear complexity, designed to approximate the optimum power control algorithm. The SRA algorithm in each step achieves C/I balance. If the achieved C/I level, γ^*, is not sufficient, one transmitter is shut off (removed) according to some simple removal criterion. The system is rebalanced and cell removals continue until the required C/I level, γ_0, is reached in the reduced system. Independent power control would be used in the up- and down-links. The SRA algorithm uses full knowledge of the link gain matrix in order to calculate its eigenvalues. To be able to use the algorithm in an environment where only the local average C/I values are known we now modify the SRA algorithm by using the DB algorithm introduced in the previous section. We therefore propose the following algorithm.

Limited Information SRA-Algorithm (LI-SRA)

1) Set $\boldsymbol{P} = \boldsymbol{1}$. Measure and store the C/I vector $\Gamma^{(0)}$. If $\Gamma_i^{(0)} > \gamma_0$ for all i, stop; otherwise,
2) Operate the distributed balancing algorithm for at most L steps. If at some step $\nu(\nu < L)$, $\Gamma_i^{(\nu)} > \gamma_0$ for all i, stop; otherwise
3) Remove the cell i that has the smallest initial C/I, $\Gamma_i^{(0)}$. Go to step 1.

The power vector is reset to equal power in all cells after each cell removal. The removal criterion removes the cell with the smallest initial C/I. After the cell removal, we reapply the DB algorithm on the square submatrix of $\boldsymbol{Z}$ corresponding to the removal of row i and column i.

A critical parameter for the algorithm is L, the maximum number of iterations we allow for balancing. During the balancing period the minimum (achieved) C/I level will increase gradually. As we will see in Section V, the balancing algorithm does not guarantee that the C/I's for all links approach γ^* in a monotone fashion. This means that a link with initially sufficient C/I may, during the process of balancing, drop below the quality threshold. The link may stay there, and later in the process return to the C/I level above the threshold.

Since we have no prior knowledge of γ^*, using the DB algorithm for too long a time involves a certain risk. If $\gamma^* > \gamma_0$ we should keep balancing since we, eventually, will reach the required C/I in all links. On the other hand, in the case where $\gamma^* < \gamma_0$ we will end up with unacceptable interference in *all* links. In the latter case we should proceed to step 3) quickly and remove a cell. The proper selection of L will be a tradeoff between these risks. In addition, we have to take into account that the link gains are constantly changing in a mobile radio environment. The speed of change of the matrix $\boldsymbol{Z}$ will therefore provide an upper limit to L. The latter problem is further discussed in the following section.

We have to note that the LI-SRA algorithm still, in a sense, is a global algorithm. The balancing procedure, which is the computationally most intensive one, is almost completely distributed. Only now and then we would have to rescale the power levels (13) in order to maintain a reasonable dynamic range requirement for the transmitter power. The removal procedure, however, requires the collection of data from the cells in order to compare the C/I values in the different cells. This is a straightforward procedure in a global (network) control scheme. As has been noted in [5], [6], the removal procedure is already part of the normal handoff/dynamic assignment procedures of the system. Therefore, this power control scheme should not impose any significant additional computational burden on the system. A completely distributed implementation would, however, require some means of sharing of the C/I data which, on the other hand, may not be a simple problem.

V. C/I Estimation Errors

As we have briefly noted in Section II, the link gain matrices in a real cellular system are constantly changing due to the movements of the mobiles. Our assumption was that if the iteration in DB algorithm is performed "fast' enough, however, the matrix $\boldsymbol{Z}$ may be regarded as constant. Now, let us investigate in somewhat more detail, which of these requirements actually apply. For this purpose, let us assume that our transmission system has some means of coping with the small-scale, multipath fading in the channel (channel coding, equalization, diversity, etc.). Further, we will assume that the typical time of convergence of our power control procedure is much smaller than the typical duration of a call. In this case, the large-scale propagation effects and shadowing fading would be the main causes of the variations in $\boldsymbol{Z}$. The C/I requirement would be a requirement on the local average C/I level, not the instantaneous C/I level. Thus we may regard $\boldsymbol{Z}$ as constant, if the algorithm reaches an adequate solution to the eigenvalue problem within the typical coherence time of the (log-normal) shadow fading process. In a cellular mobile radio system this time is usually on the order of 1–10 s. On the other hand, in each step, the local average C/I has to be estimated with some degree of reliability.

TABLE I
TYPICAL EXPECTED NUMBER OF ITERATIONS FOR APPROXIMATELY CONSTANT LINK GAIN MATRIX Z
(Case a) corresponds to small cell urban system, case b) to microcellular urban system. $f = 900$ MHz.)

	Mobile Speed (m/s)	Coh. Time (s)	Av. Time (s)	No. of Iterations
a)	1 (hand-portable)	200	6	30
	20	10	0.3	30
b)	1 (hand-portable)	50	6	8
	20	2.5	0.3	8

These measurements are corrupted by the multipath fading pattern which typically extends over several wavelengths. This will, in turn, determine an upper limit to the iteration rate. Table I shows some typical values for the expected number of iteration steps that could be achieved for different vehicle speeds when applying standard propagation models for mobile communications [8], [11].

In almost all cases, the estimated C/I value will more or less differ from the "actual" C/I level due to either the slow estimation process or the limited suppression of multipath fading or measurement noise. In order to analyze the effect of these estimation errors, we use the following simple model. Let the estimated C/I, $\Gamma_i^{(\nu)}$, used by the DB algorithm be computed as

$$\Gamma_i^{\prime(\nu)} = \eta_i^{(\nu)} \Gamma_i^{(\nu)} \tag{14}$$

where $\eta_i^{(\nu)}$ are all independent log-normal random variable with log-variance σ_m. This corresponds to letting the C/I levels, measured in decibels, be corrupted by additive, zero mean, Gaussian variable with variance σ_m. Since the algorithm converges for all allowed (positive) power vectors $\boldsymbol{P}$ one is tempted to believe that the algorithm is rather insensitive to small changes in $\boldsymbol{Z}$ and will "track" the presently achievable C/I with reasonable accuracy. The numerical results in the following section corroborate this suggestion.

VI. NUMERICAL RESULTS

A. Simulation Model

To derive comparable numerical results, we use the same model as in [6]. We consider a fixed and symmetric channel assignment strategy that divides the cells in K different channel groups. The cells using the same frequency are placed symmetrically in a hexagonal grid. Base stations use omnidirectional antennas and are located at the center of the cells. The locations of the mobiles are uniformly distributed over the cell area. We assume that all link gains are affected by shadow fading. The average signal power is assumed to decrease with the fourth power of the distance [8], [11]. The link gains may thus be expressed as

$$G_{ij} = \frac{A_{ij}}{d_{ij}^4} \tag{15}$$

where d_{ij} is the distance between base station j and mobile i and the stochastic variables A_{ij} are independent log-normal variables, where

$$E[10 \log A_{ij}] = 0 \tag{16a}$$
$$Var[10 \log A_{ij}] = \sigma^2. \tag{16b}$$

All the numerical results presented in the following are for a 4-by-4 ($Q = 16$), "square," cochannel cell pattern. All cochannels in the set are assumed to be in use. Interference probabilities are evaluated by Monte Carlo simulations for 1000 independent configurations (link-gain matrices). The standard deviation of the interference probability estimates should, therefore, except for extremely small values of γ, be in the order of 1%. Unless otherwise noted, log-normal fading with log variance $\sigma = 6$ dB is assumed.

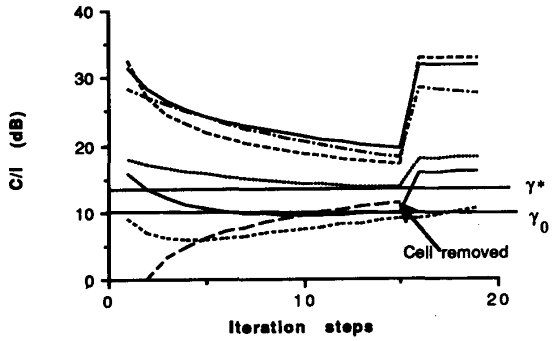

Fig. 2. Typical convergence of the LI-SRA algorithm. The graph shows the C/I levels as function of the number of algorithm steps for 7 of the 16 cells. The required minimum C/I is 10 dB, the maximum achievable C/I is approximately 13 dB. The cell with the lowest *initial* C/I is removed after $L = 15$ steps. $K = 7$.

B. LI-SRA Algorithm Performance

Fig. 2 illustrates typical behavior of the LI-SRA algorithm. The graph shows an example of the C/I levels as function of time in a few selected cells in a typical cochannel set. The required γ_0 is here set to 10 dB, whereas the maximum achievable C/I for the cochannel set γ^* happens to be about 13 dB. We make the rather pessimistic assumption that the system is initially unbalanced ($\boldsymbol{P}_0 = 1$). As we can see, the C/I trajectories immediately start to converge towards γ^*. After $L = 15$ steps, the removal algorithm is invoked. The initially "worst" cell is removed, the powers are reset to a constant level, and the balancing process is resumed. Note that this cell does not have the worst C/I level at the point of removal(!). After 18 steps, all remaining links finally reach the required minimum threshold and the algorithm stops.

An alternative removal algorithm, that removes the cell with the worst "final" C/I level after balancing, has also been investigated. This procedure yields comparable results [12]. Similar results are also obtained when using the resulting power vector immediately before the cell removal as the starting vector in the next balancing process. However, the latter procedure achieves smoother C/I trajectories and may, in some cases, speed up the algorithm. This alternative balancing algorithm may be particularly useful in a perhaps more common case, where we add one new cell to an

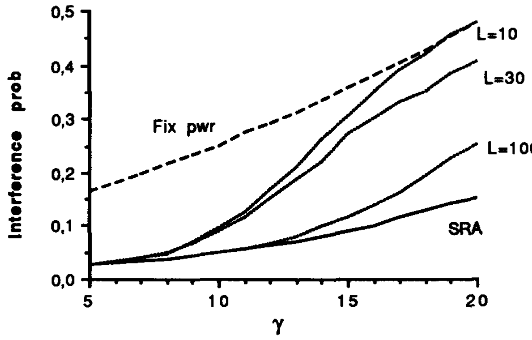

Fig. 3. Interference probability for LI-SRA algorithm for various values of the maximum number of balancing steps L, compared to the performance of the SRA algorithm. $K = 7$.

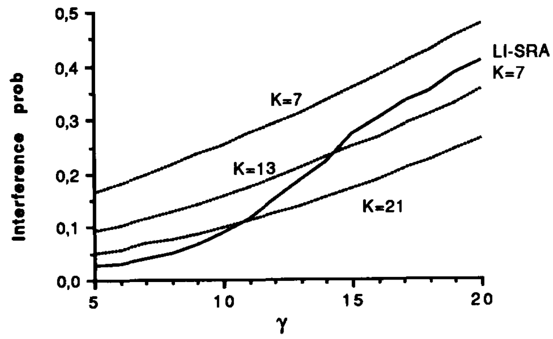

Fig. 5. Interference probability for LI-SRA algorithm for $K = 7$ compared to fixed power systems for cluster sizes $K = 7$, 13, and 21. $L = 30$.

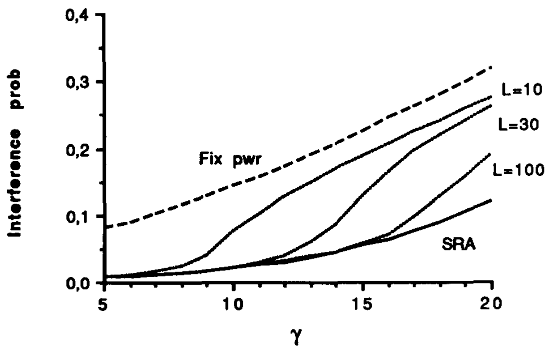

Fig. 4. Interference probability for LI-SRA algorithm for various values of the maximum number of balancing steps L, compared to the performance of the SRA algorithm. $K = 13$.

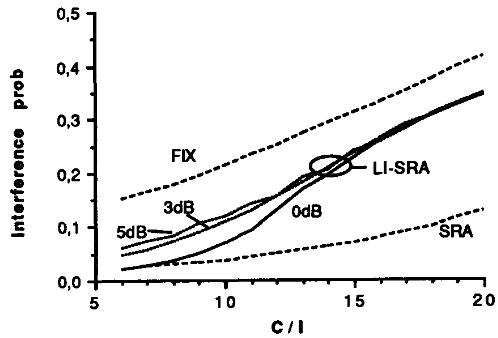

Fig. 6. Interference probability for LI-SRA algorithm for $K = 7$ with C/I-estimation errors compared to fixed power systems and a system with ideal SRA power control. $\sigma_m = 0$, 3, and 5 dB, $L = 30$.

already balanced system. This should be a topic for further investigations.

Figs. 3 and 4 compare the performance of the LI-SRA algorithm to the performance of the original SRA for various values of the maximum number of balancing steps L. As we can see, the LI-SRA algorithm provides a good approximation for low and moderate values of γ_0. As we may expect from (12), the convergence is slow for high C/I's. Thus to achieve a reasonable approximation of the SRA, a fairly large number of balancing steps are required. Fig. 5 provides a capacity comparison. A system using the LI-SRA algorithm of cluster size $K = 7$ is compared to fixed power systems with cluster sizes $K = 7$, 13, and 21. At the 10% interference probability level, we see that the fixed system requires a cluster size $K = 21$ to equal the LI-SRA system. We may interpret this as a capacity gain of roughly 3.

Fig. 6, finally, illustrates the impact of estimation errors as modeled by (14). The independent log-normal errors η_i are here assumed to have a log-variance σ_m of 3 and 5 dB. The noise will keep the system from reaching a perfect C/I balance. As expected, the measurement errors introduce a slight loss in performance for low protection ratios. In our example, the number of balancing steps is too low ($L = 30$) to handle high C/I requirements. As a result, we see that for moderate C/I's the loss due to estimation errors is negligible compared to the performance loss due to poor C/I balancing.

VII. Discussion

Throughout the paper we have studied the properties of distributed power control algorithms. The main implementational drawback of the optimum algorithm described in [6] was the vast amount of path-gain measurements required to obtain the optimum power vector. In practice, it would be impractical to attempt to estimate all these parameters. We have, however, shown that a limited-information algorithm, the LI-SRA, using only measurements of the signal-to-interference ratio in the *wanted* links, can achieve results that are comparable to the SRA scheme. Although some performance is lost compared to the "full-information" algorithm, capacity gains on the order of 3–4 are still feasible for systems with slowly varying link gains.

However, in our analysis we have assumed the path-gain matrix to be roughly constant during the balancing phase of the algorithm. The rate of change in the link-gain matrix due to shadowing is therefore a crucial issue. For proper convergence we need an estimate of the local average C/I at a high rate, whereas the quality of these estimates decreases as we decrease the averaging intervals. From the discussion in Section V, we may assume that an implementation of the algorithm, from this point view, would work very well in the traditional cellular system with large or moderate cell sizes. For very small cell sizes (urban street microcells) and indoor systems with rapid changes in propagation, the tradeoff between measurement accuracy and algorithm speed would have to be studied more carefully. The numerical results, however, indicate that the LI-SRA scheme is robust to measurement error and should therefore be useful even if the C/I measurements were slow and less accurate. The same property should also be useful in handling transients as new calls are introduced or when existing calls are terminated. However, thus far no explicit studies have been performed regarding the latter topic.

When evaluating the performance of the balancing algorithm, we were considering the rather difficult case where, initially, the entire cochannel set was unbalanced. A topic for further research would be to investigate the much more common case where only one or a few cells are added to an already balanced system.

Finally, it may be argued that the LI-SRA algorithm is not completely distributed. This is of course true. Although the computationally most intensive part, the C/I balancing, is distributed, the selection of which cell to "remove" still requires some global, or at least regional, coordination. Since "cell removals" usually correspond to intracell handoffs, proper coordination of the "removal" process with the handoff and the dynamic channel allocation procedures of the system would ensure that no additional communication capacity would be required.

REFERENCES

[1] J. M. Aein, "Power balancing in systems employing frequency reuse," *COMSAT Tech. Rev.*, vol. 3, no. 2, Fall 1973.

[2] R. W. Nettleton and H. Alavi, "Power control for spread-spectrum cellular mobile radio system," in *Proc. IEEE Veh. Tech. Conf.*, VTC-83, 1983, pp. 242–246.

[3] H. Axén, "Power control in cellular mobile telephone systems," Diploma thesis, Ericsson Radio Systems, Int. Rep. BT/SU 90:1708, 1990 (in Swedish).

[4] ——, "Uplink C/I as control parameter for mobile station power control," Ericsson Radio Systems, Int. Rep., BT/ST 90:2377, 1990 (in Swedish).

[5] J. Zander, "Optimum global transmitter power control in cellular radio systems," in *Proc. IEEE Int. Symp. on Personal, Indoor and Mobile Radio Commun.*, London, U.K., Sept. 1991.

[6] ——, "Performance of optimum transmitter power control in cellular radio systems,"*IEEE Trans. Veh. Technol.*, vol. 41, pp. 57–62, Feb. 1992

[7] F. R. Gantmacher, *The Theory of Matrices*, vol. 2. New York: Chelsea, 1974, ch. XIII.

[8] W. C. Y. Lee, "Elements of cellular mobile radio," *IEEE Trans. Veh. Technol.*, vol. VT-35, May 1986.

[9] R. Prasad, A. Kegel, and J. Arnbak, "Analysis of system performance of high-capacity mobile radio," *Proc. IEEE Veh. Technol. Conf, VTC-89*, 1989, pp. 306–310.

[10] B. Carnahan, H. A. Luther, and J. O. Wilkes, *Applied Numerical Analysis*. New York: Wiley, 1969.

[11] W. C. Jakes, Ed., *Microwave Mobile Communications*. New York: Wiley, 1976.

[12] J. Zander, "Near-optimum transmitter power control in cellular radio systems—Implementational considerations," Int. Rep. Royal Inst. Technol., TRITA-TTT-9108, Apr. 1991.

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